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ADVECTION/DIFFUSION OF LARGE-SCALE B-FIELD IN ACCRETION DISKS

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ABSTRACT

Activity of the nuclei of galaxies and stellar mass systems involving disk accretion to black holes is thought to be due to (1) a small-scale turbulent magnetic field in the disk (due to the magneto-rotational instability or MRI) which gives a large viscosity enhancing accretion, and (2) a large-scale magnetic field which gives rise to matter outflows and/or electromagnetic jets from the disk which also enhances accretion. An important problem with this picture is that the enhanced viscosity is accompanied by an enhanced magnetic diffusivity which acts to prevent the build up of a significant large-scale field. Recent work has pointed out that the disk's surface layers are non-turbulent and thus highly conducting (or non-diffusive) because the MRI is suppressed high in the disk where the magnetic and radiation pressures are larger than the thermal pressure. Here, we calculate the vertical (z) profiles of the stationary accretion flows (with radial and azimuthal components), and the profiles of the large-scale, magnetic field taking into account the turbulent viscosity and diffusivity due to the MRI and the fact that the turbulence vanishes at the surface of the disk. We derive a sixth-order differential equation for the radial flow velocity $v_r(z)$ which depends mainly on the midplane thermal to magnetic pressure ratio $\beta > 1$ and the Prandtl number of the turbulence $\mathcal{P}$ = viscosity/diffusivity. Boundary conditions at the disk surface take into account a possible magnetic wind or jet and allow for a surface current in the highly conducting surface layer. The stationary solutions we find indicate that a weak ($\beta > 1$) large-scale field does not diffuse away as suggested by earlier work. For a wide range of parameters $\beta > 1$ and $\mathcal{P} \geq 1$, we find stationary *channel* type flows where the flow is radially *outward* near the midplane of the disk and radially *inward* in the top and bottom parts of the disk. Channel flows with inward flow near the midplane and outflow in the top and bottom parts of the disk are also found. We find that Prandtl numbers larger than a critical value (estimated to be 2.7) are needed in order for there to be magnetocentrifugal outflows from the disk's surface. For smaller $\mathcal{P}$, electromagnetic outflows are predicted.

Subject headings: accretion, accretion disks — galaxies: jets — magnetic fields — MHD — X-rays: binaries

1. INTRODUCTION

Early work on disk accretion to a black hole argued that a large-scale poloidal magnetic field originating from say the interstellar medium, would be dragged inward and greatly compressed near the black hole by the accreting plasma (Bisnovatyi-Kogan & Ruzmaikin 1974, 1976) and that this would be important for the formation of jets (Lovelace 1976). Later, the importance of a weak small-scale magnetic field within the disk was recognized as the source of the turbulent viscosity of disk owing to the magneto-rotational instability (MRI; Balbus & Hawley 1991, 1998). Analysis of the diffusion and advection of a large-scale field in a disk with a turbulent viscosity comparable to the turbulent magnetic diffusivity (as suggested by MRI simulations) indicated that a *weak* large-scale field would diffuse outward rapidly (van Ballegooijen 1989; Lubow, Papaloizou, & Pringle 1994; Lovelace, Romanova, & Newman 1994, 1997). This has led to the suggestion that special conditions (nonaxisymmetry) are required for the field to be advected inward (Spruit & Uzdensky 2005). Recently, Bisnovatyi-Kogan and Lovelace (2007) pointed out that the disk's surface layers are highly conducting (or non-diffusive) because the MRI is suppressed in this region where the magnetic energy-density is larger than the thermal energy-density. Rothstein and Lovelace (2008) analyzed this problem in further detail and discussed the connections with global and shearing box magnetohydrodynamic (MHD) simulations of the MRI.

With the disk's highly conducting surface neglected, the fast outward diffusive drift of the large-scale poloidal magnetic field in a turbulent disk can be readily understood by looking at the induction equation for the vertical field B_z,

$$\frac{\partial(rB_z)}{\partial t} = \frac{\partial}{\partial r}\left[(rB_z)u - \eta r\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\right] ,$$

where $u > 0$ is the accretion speed and η is the turbulent diffusivity both assumed independent of z (Lovelace et al. 1994). (Both assumptions are invalid and are removed in this work.) For a dipole type field, B_z is an even function of z and it changes very little over the half-thickness h of a thin disk ($h \ll r$). Consequently, the average of this equation over the half-thickness gives

$$\frac{\partial(rB_z)}{\partial t} = \frac{\partial}{\partial r}\left[(rB_z)u - \left(\frac{r}{h}\right)\eta(B_r)_h + \eta r\frac{\partial B_z}{\partial r}\right] ,$$

where $(B_r)_h$ is the radial field at the disk's surface. The terms inside the square brackets represents the flux of the $z-$magnetic flux. For stationary conditions it vanishes. The term proportional to u describes the *inward advection* of the magnetic field, the term $\propto \eta(B_r)_h$ describes

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the *outward diffusive drift* of the field, and the last term represents the *radial diffusion* of the field. For $(B_r)_h \sim B_z$ the radial diffusion term is negligible compared with the diffusive drift term. For a weak B_z field and the standard $\alpha-$disk model with turbulent viscosity of the order of the turbulent diffusivity, the accretion speed is $u \sim \alpha c_s h/r$ (with c_s the sound speed), whereas the outward drift speed of the field is $\sim \alpha c_s$ which is a factor $r/h \gg 1$ larger than u. That is, a stationary solution with $B_z \neq 0$ is not possible. A stationary or growing B_z field is possible if the field is sufficiently strong to give appreciable outflow of angular momentum to jets (Lovelace et al. 1994).

Three-dimensional MHD simulations have been performed which give some information on the advection/diffusion of a large scale field. These simulations resolve the largest scales of the MRI turbulence and therefore self-consistently include the turbulent viscosity and diffusivity. Most simulations performed to date have investigated conditions in which the accreting matter contains no net magnetic flux and where no magnetic field is supplied at the boundary of the computational domain (e.g., Hirose et al. 2004; De Villiers et al. 2005; Hawley & Krolik 2006; McKinney & Narayan 2007). In these simulations stretching of locally poloidal field lines in the initial configuration leads to a large-scale poloidal fields and jet structures in the inner disk. However, in simulations by Igumenshchev, Narayan, & Abramowicz 2003; Igumenshchev 2008) weak poloidal flux injected at the outer boundary is clearly observed to be dragged into the central region of the disk, leading to the buildup of a strong poloidal magnetic field close to the central object. This flux build up and its limit are discussed by Narayan, Igumenshchev, & Abramowicz (2003). The extent to which the magnetic field advection seen in numerical simulations depends on having a thick disk or nonaxisymmetric conditions is unclear.

In this work we calculate the profiles through the disk of stationary accretion flows (with radial and azimuthal components), and the profiles of a large-scale, weak magnetic field taking into account the turbulent viscosity and diffusivity due to the MRI and the fact that the turbulence vanishes at the surface of the disk. By a weak field we mean that the magnetic pressure in the middle of the disk is less than the thermal pressure. Related calculations of the disk structure were done earlier by Königl (1989), Li (1995), Ogilvie and Livio (2001) but without taking into account the absence of turbulence at the disk's surface. Recent work calls into question the α-description of the MRI turbulence in accretion disks and develops a closure model which fits shearing box simulation results (Pessah, Chan, & Psaltis 2008). Analysis of this more complicated model is deferred a future study.

Section 2 develops the equations for the flow and magnetic field in a viscous diffusive disk. Section 3 discusses the boundary conditions at the surface of the disk. Section 4 derives the internal flow/field solutions for an analytically soluble disk model. Section 5 discusses the external flow/field solutions which may be magnetocentrifugal winds or electromagnetic outflows. Section 6 gives the conclusions of this work.

2. THEORY

We consider the non-ideal magnetohydrodynamics of a thin axisymmetric, viscous, resistive disk threaded by a large-scale dipole-symmetry magnetic field $\mathbf{B}$. We use a cylindrical (r, ϕ, z) inertial coordinate system in which the time-averaged magnetic field is $\mathbf{B} = B_r\hat{\mathbf{r}} + B_\phi\hat{\boldsymbol{\phi}} + B_z\hat{\mathbf{z}}$, and the time-averaged flow velocity is $\mathbf{v} = v_r\hat{\mathbf{r}} + v_\phi\hat{\boldsymbol{\phi}} + v_z\hat{\mathbf{z}}$. The main equations are

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla p + \rho\mathbf{g} + \frac{1}{c}\mathbf{J}\times\mathbf{B} + \mathbf{F}^\nu \ , \tag{1}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \nabla\times(\eta\nabla\times\mathbf{B}) \ . \tag{2}$$

These equations are supplemented by the continuity equation, $\nabla(\rho\mathbf{v}) = 0$, by $\nabla\times\mathbf{B} = 4\pi\mathbf{J}/c$, and by $\nabla\cdot\mathbf{B} = 0$. Here, η is the magnetic diffusivity, $\mathbf{F}^\nu = -\nabla\cdot T^\nu$ is the viscous force with $T^\nu_{jk} = -\rho\nu(\partial v_j/\partial x_k + \partial v_k/\partial x_j - (2/3)\delta_{jk}\nabla\cdot\mathbf{v})$ (in Cartesian coordinates), and ν is the kinematic viscosity. For simplicity, in place of an energy equation we consider the adiabatic dependence $p \propto \rho^\gamma$, with γ the adiabatic index.

We assume that both the viscosity and the diffusivity are due to magneto-rotational (MRI) turbulence in the disk so that

$$\nu = \mathcal{P}\eta = \alpha\ \frac{c_{s0}^2}{\Omega_K}\ g(z) \ , \tag{3}$$

where $\mathcal{P}$ is the magnetic Prandtl number of the turbulence assumed a constant of order unity (Bisnovatyi-Kogan & Ruzmaikin 1976), $\alpha \leq 1$ is the dimensionless Shakura-Sunyaev (1973) parameter, c_{s0} is the midplane isothermal sound speed, $\Omega_K \equiv (GM/r^3)^{1/2}$ is the Keplerian angular velocity of the disk, and M is the mass of the central object. The function $g(z)$ accounts for the absence of turbulence in the surface layer of the disk (Bisnovatyi-Kogan & Lovelace 2007; Rothstein & Lovelace 2008). In the body of the disk $g = 1$, whereas at the surface of the disk, at say z_S, g tends over a short distance to a very small value $\sim 10^{-8}$, effectively zero, which is the ratio of the Spitzer diffusivity of the disk's surface layer to the turbulent diffusivity of the body of the disk. At the disk's surface the density is much smaller than its midplane value.

We consider stationary solutions of equations (1) and (2) for a weak large-scale magnetic field. These can be greatly simplified for thin disks where the disk half-thickness, of the order of $h \equiv c_{s0}/\Omega_K$, is much less than r. Thus we have the small parameter

$$\varepsilon = \frac{h}{r} = \frac{c_{s0}}{v_K} \ll 1 \ . \tag{4}$$

It is useful in the following to use the dimensionless height $\zeta \equiv z/h$.

The three magnetic field components are assumed to be of comparable magnitude on the disk's surface, but $B_r = 0 = B_\phi$ on the midplane. On the other hand the axial magnetic field changes by only a small almount going from the midplane to the surface, $\Delta B_z \sim \varepsilon B_r \ll B_z$ (from $\nabla\cdot\mathbf{B} = 0$) so that $B_z \approx$ const inside the disk. As a consequence, the $\partial B_j/\partial r$ terms in the magnetic force in equation (1) can all be dropped in favor of the $\partial B_j/\partial z$ terms (with $j = r,\ \phi$). Thus, we neglect the final term of the induction equation given in the Introduction. It is important to keep in mind that B_j is the large scale field; the approximation does not apply to the small-scale field

which gives the viscosity and diffusivity. The three velocity components are assumed to satisfy $v_z^2 \ll c_{s0}^2$ and $v_r^2 \ll v_\phi^2$. Consequently, $v_\phi(r,z)$ is close in value to the Keplerian value $v_K(r) \equiv (GM/r)^{1/2}$. Thus, $\partial v_\phi/\partial r = -(1/2)(v_\phi/r)$ to a good approximation.

With these assumptions, the radial component of equation (1) gives

$$\rho\left(\frac{GM}{r^2} - \frac{v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r} + \frac{1}{4\pi}B_z\frac{\partial B_r}{\partial z} + F_r^\nu \tag{5}$$

The dominant viscous force is $F_r^\nu = -\partial T_{rz}^\nu/\partial z$ with $T_{rz}^\nu = -\rho\nu\partial v_r/\partial z$.

We normalize the field components by $B_0 = B_z(r, z = 0)$, with $b_r = B_r/B_0$, $b_\phi = B_\phi/B_0$, and $b_z = B_z/B_0 \approx 1$. Also, we define $u_\phi \equiv v_\phi(r,z)/v_K(r)$ and the accretion speed $u_r \equiv -\, v_r/(\alpha c_{s0})$. For the assumed dipole field symmetry, b_r and b_ϕ are odd functions of ζ whereas u_r and u_ϕ are even functions.

Equation (5) then gives

$$\frac{\partial b_r}{\partial\zeta} = \frac{\beta\tilde\rho}{\varepsilon}\ \left(1 - k_p\ \varepsilon^2 - u_\phi^2\right) + \alpha^2\beta\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial u_r}{\partial\zeta}\right)\ , \tag{6}$$

where $\tilde\rho \equiv \rho(r,z)/\rho_0$ with $\rho_0 \equiv \rho(r, z = 0)$. The midplane plasma beta is

$$\beta \equiv \frac{4\pi\rho_0 c_{s0}^2}{B_0^2}\ , \tag{7}$$

where $k_p \equiv -\partial\ln p/\partial\ln r$ is assumed of order unity and $p = \rho c_s^2$. Note that $\beta = c_{s0}^2/v_{A0}^2$, where $v_{A0} = B_0/(4\pi\rho_0)^{1/2}$ is the midplane Alfvén velocity. The rough condition for the MRI instability and the associated turbulence in the disk is $\beta > 1$ (Balbus & Hawley 1998). In the following we assume $\beta > 1$, which we refer to as a weak magnetic field.

The $\phi-$component of equation (1) gives

$$\frac{\partial b_\phi}{\partial\zeta} = \frac{\alpha\beta\tilde\rho}{2}\ \left(3\varepsilon k_\nu g - u_r\right) - \frac{\alpha\beta}{\varepsilon}\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial u_\phi}{\partial\zeta}\right)\ , \tag{8}$$

where $k_\nu \equiv \partial\ln(\rho c_{s0}^2 r^2/h)/\partial\ln(r) > 0$ is of order unity. In addition to the well-know viscous force $[F_\phi^\nu(a) = -r^{-2}\partial(r^2 T_{r\phi}^\nu)/\partial r$ with $T_{r\phi}^\nu = -\rho\nu r\partial(v_\phi/r)/\partial r]$ which gives the term $\propto k_\nu$, we must include the force contribution $F_\phi^\nu(b) = -\partial T_{\phi z}^\nu/\partial z$ with $T_{\phi z}^\nu = -\rho\nu\partial v_\phi/\partial z$. This gives the second derivative term in equation (8).

Note that integration of equation (8) from $\zeta = 0$ (where $b_\phi = 0$ and $\partial u_\phi/\partial\zeta = 0$) to $\zeta_S + \epsilon$ (where $g = 0$) gives

$$b_{\phi S+} = \frac{1}{2}\alpha\beta\tilde\Sigma\left(3\varepsilon k_\nu - \overline{u}_r\right)\ ,$$

where $\overline{u}_r \equiv \int_0^{\zeta_S} d\zeta\ \tilde\rho u_r/\tilde\Sigma$ is the average accretion speed, $\tilde\Sigma \equiv \int_0^{\zeta_S} d\zeta\ \tilde\rho$, and the $S+$ subscript indicates evaluation at $\zeta = \zeta_S + \epsilon$. The average accretion speed, written as

$$\overline{u}_r = u_0 - \frac{2b_{\phi S+}}{\alpha\beta\tilde\Sigma}\ , \tag{9}$$

is the sum of a viscous contribution, $u_0 \equiv 3\varepsilon k_\nu$, and a magnetic contribution ($\propto b_{\phi S+}$) due to the loss of angular momentum from the surface of the disk where necessarily $b_{\phi S+} \leq 0$ (Lovelace, Romanova, & Newman 1994). Equation (9) is discussed further in §5. The continuity equation implies that $rh\rho_0\tilde\Sigma(\alpha c_{s0}\overline{u}_r)$ is independent of r.

The $z-$component of equation (1) gives

$$\frac{\partial p}{\partial\zeta} = -\rho c_{s0}^2\zeta - \frac{\rho_0 c_{s0}^2}{2\beta}\frac{\partial}{\partial\zeta}\left(b_r^2 + b_\phi^2\right)\ . \tag{10}$$

The neglected viscous force in this equation, $r^{-1}\partial(rT_{rz}^\nu)/\partial r$ with $T_{rz} = -\rho\nu\partial v_r/\partial z$, is smaller than the retained pressure gradient term by a factor of order $\alpha^2\varepsilon \ll 1$. The term involving the magnetic field describes the magnetic compression of the disk because $b_r^2 + b_\phi^2$ at the surface of the disk is larger than its midplane value which is zero (Wang, Sulkanen, & Lovelace 1990). For $\beta \gg 1$ the compression effect is small and can be neglected.

As mentioned we assume $p \propto \rho^\gamma$ which can be written as $p = \rho c_{s0}^2(\rho/\rho_0)^{\gamma-1}$. Thus

$$\tilde\rho = \frac{\rho}{\rho_0} = \left(1 - \frac{(\gamma-1)\zeta^2}{2\gamma}\right)^{1/(\gamma-1)}\ , \tag{11}$$

for $\beta \gg 1$. The density goes to zero at $\zeta_m = [2\gamma/(\gamma-1)]^{1/2}$. However, before this distance is reached the MRI turbulence is suppressed, and $g(\zeta)$ in equation (3) is effectively zero.

The toroidal component of Ohm's law (equivalent to equation 2), $J_\phi = \sigma(\mathbf{v}\times\mathbf{B})_\phi$, with $\sigma = c^2/(4\pi\eta)$, gives

$$\frac{\partial b_r}{\partial\zeta} = \frac{\mathcal{P}}{g}u_r\ . \tag{12}$$

Multiplying this equation by g, integrating from $\zeta_S - \epsilon$ to $\zeta_S + \epsilon$, and using the facts that $\partial g/\partial\zeta = -\delta(\zeta - \zeta_S)$ and that $|u_r|$ is bounded implies that $b_{rS+} = b_{rS-}$. That is, there is no jump in b_r across the highly conducting surface layer.

Note that multiplying equation (12) by g and integration from $\zeta = 0$ to $\zeta_S + \epsilon$ gives

$$b_{rS} = \mathcal{P}\zeta_S\langle u_r\rangle\ , \tag{13}$$

where $\langle..\rangle = \int_0^{\zeta_S} d\zeta(..)/\zeta_S$.

The other components of Ohm's law give

$$\frac{\partial u_\phi}{\partial\zeta} = \frac{3\varepsilon}{2}b_r - \frac{\alpha\varepsilon}{\mathcal{P}}\frac{\partial}{\partial\zeta}\left(g\frac{\partial b_\phi}{\partial\zeta}\right)\ . \tag{14}$$

Combining equations (6) and (12) gives

$$u_r = \frac{\beta\tilde\rho g}{\varepsilon\mathcal{P}}\left(1 - k_p\varepsilon^2 - u_\phi^2\right) + \frac{\alpha^2\beta g}{\mathcal{P}}\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial u_r}{\partial\zeta}\right)\ . \tag{15}$$

For thin disks, $\varepsilon \ll 1$, and $\beta > 1$, we have $u_\phi = 1 + \delta u_\phi$ with $(\delta u_\phi)^2 \ll 1$ which follows from the integral of equation (6). Consequently,

$$\delta u_\phi = -\frac{k_p\varepsilon^2}{2} - \frac{\varepsilon\mathcal{P}u_r}{2\beta\tilde\rho g} + \frac{\alpha^2\varepsilon}{2\tilde\rho}\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial u_r}{\partial\zeta}\right)\ , \tag{16}$$

to a good approximation.

We first take the derivative of equation (14) and then substitute the b_r derivative with equation (12). In turn, the u_ϕ derivatives can be put in terms of u_r and its derivatives using equation(16). In this way we obtain

$$\begin{aligned}
&\alpha^4\beta^2\frac{\partial^2}{\partial\zeta^2}\left(g\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial}{\partial\zeta}\left(\frac{1}{\tilde\rho}\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial u_r}{\partial\zeta}\right)\right)\right)\right)\\
&\quad - \alpha^2\beta\mathcal{P}\frac{\partial^2}{\partial\zeta^2}\left(g\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial}{\partial\zeta}\left(\frac{u_r}{\tilde\rho g}\right)\right)\right)\\
&\quad - \alpha^2\beta\mathcal{P}\frac{\partial^2}{\partial\zeta^2}\left(\frac{1}{\tilde\rho}\frac{\partial}{\partial\zeta}\left(\tilde\rho g\frac{\partial u_r}{\partial\zeta}\right)\right)\\
&\quad + \alpha^2\beta^2\frac{\partial^2}{\partial\zeta^2}\left(\tilde\rho g\left(u_r - gu_0\right)\right) + \mathcal{P}^2\frac{\partial^2}{\partial\zeta^2}\left(\frac{u_r}{\tilde\rho g}\right)\\
&\quad + 3\beta\mathcal{P}^2\frac{u_r}{g} = 0\ .
\end{aligned} \tag{17}$$

The equation can be integrated from $\zeta = 0$ out to the surface of the disk ζ_S where boundary conditions apply. Because u_r is an even function of ζ, the odd derivatives of u_r are zero at $\zeta = 0$ and one needs to specify $u_r(0)$, $u_r''(0)$, and $u_r^{iv}(0)$. A "shooting method" can be applied where the values of $u_r(0)$, $u_r''(0)$, and $u_r^{iv}(0)$ are adjusted to satisfy the boundary conditions. Once $u_r(\zeta)$ is calculated, equations (8), (12), and (14) can be integrated to obtain $b_\phi(\zeta)$, $b_r(\zeta)$, and $u_\phi(\zeta)$.

For specificity we take

$$g(\zeta) = \left(1 - \frac{\zeta^2}{\zeta_S^2}\right)^\delta , \tag{18}$$

where $\zeta_S < \zeta_m$ and $\delta \ll 1$. That is, we neglect the ratio of the Spitzer diffusivity on the surface of the disk to its value in the central part of the disk. An estimate of ζ_S can be made by noting that $\beta(\zeta) = 4\pi p(\zeta)/B_0^2 = \beta(\tilde\rho)^\gamma \approx 1$ at ζ_S. This gives $\zeta_S^2/\zeta_m^2 = 1 - \beta^{-(\gamma-1)/\gamma}$ and $\rho_S/\rho_0 = \beta^{-1/\gamma}$.

If $\mathcal{E}$ denotes the fraction of the disk mass accretion rate which goes into outflows, then we can estimate the vertical speed of matter at the disk's surface as $v_z(\zeta_S) \sim \mathcal{E}(2h/r)(\rho_0/\rho_S)|\overline{v}_r| = \mathcal{E}(2h/r)\beta^{1/\gamma}|\overline{v}_r|$. Our neglect of v_z evidently requires that $\mathcal{E}(2h/r)\beta^{1/\gamma} \ll 1$.

3. BOUNDARY CONDITIONS

We restrict our attention to physical solutions which (a) have net mass accretion,

$$\dot M = 4\pi r h \rho_0 \alpha c_{s0} \tilde\Sigma\ \overline{u}_r > 0 , \tag{19}$$

and (b) have $b_\phi \le 0$ on the disk's surface. This condition on $b_{\phi S+}$ corresponds to an efflux of angular momentum and energy (or their absence) from the disk to its corona rather than the reverse. From equation (9), the condition $b_{\phi S+} \le 0$ is the same as $\overline{u}_r \ge u_0$, where u_0 is the minimum (viscous) accretion speed. Note that $\overline{u}_r/u_0 - 1$ is the fraction of the accretion power which goes into the outflows or jets (Lovelace et al. 1994). Clearly, the condition on $b_{\phi S+}$ implies that $\dot M > 0$ so that there is only one condition.

In general there is a continuum of values of $b_{\phi S+} \le 0$ for the considered solutions inside the disk. The value of $b_{\phi S+}$ can be determined by matching the calculated fields b_{rS} and $b_{\phi S+}$ onto an external field and flow solution as discussed in §5.

From equation (12) we found that there is no jump in b_r across the conducting surface layer. Thus, integration of equation (6) from $\zeta_S - \epsilon$ to $\zeta_S + \epsilon$ implies that

$$\left.\frac{\partial u_r}{\partial \zeta}\right|_{\zeta_{S-}} = 0 . \tag{20}$$

This represents a second condition on the disk solutions.

Integration of equation (14) from $\zeta_S - \epsilon$ to $\zeta_S + \epsilon$ gives

$$u_{\phi S+} - u_{\phi S-} = \frac{\alpha\varepsilon}{\mathcal{P}} \left.\frac{\partial b_\phi}{\partial \zeta}\right|_{\zeta_{S-}} .$$

This velocity jump must be zero as can be shown by inspection of the total angular momentum flux-density in the $z-$direction, $rT_{\phi z} = -B_\phi B_z/4\pi - \rho\nu(\partial v_\phi/\partial z)$, where the first term is the magnetic stress and the second is the viscous stress. A jump in v_ϕ would give a delta-function contribution to the viscous stress which cannot be balanced by the magnetic stress. Therefore

$$\left.\frac{\partial b_\phi}{\partial \zeta}\right|_{\zeta_{S-}} = 0 . \tag{21}$$

This is a third condition on the disk solutions.

Integration of equation (8) from $\zeta_S - \epsilon$ to $\zeta_S + \epsilon$ gives

$$b_{\phi S+} - b_{\phi S-} = \frac{\alpha\beta\tilde\rho_S}{\varepsilon} \left.\frac{\partial u_\phi}{\partial \zeta}\right|_{\zeta_{S-}} . \tag{22}$$

This equation is equivalent to the continuity of the angular momentum flux-density across the surface layer, namely, $rT_{\phi z}^+ = -B_\phi^+ B_z/4\pi$ (above the layer) is equal to $rT_{\phi z}^- = -B_\phi^- B_z/4\pi - \rho\nu(\partial v_\phi/\partial z)|_-$ (below the layer). The jump $b_{\phi S+} - b_{\phi S-}$ corresponds to a radial surface current flow in the highly conducting surface layer of the disk, $\mathcal{J}_r = -(c/4\pi)B_0(b_{\phi S+} - b_{\phi S-})$.

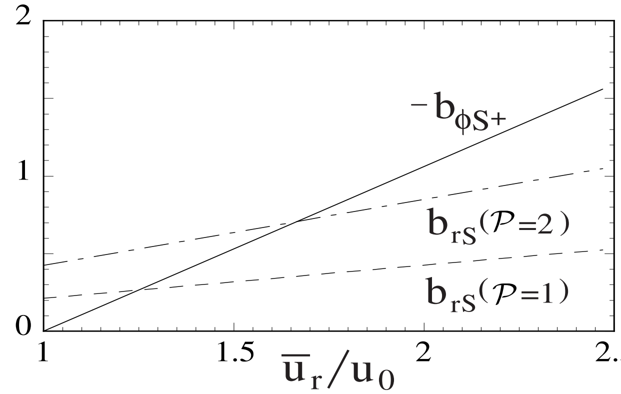

FIG. 1.— Radial and toroidal field components (normalized to B_z) at the disk's surface as a function of the average accretion speed $\overline{u}_r$ (normalized by the viscous accretion speed u_0). For this plot $\beta = 100$ and Prandtl numbers $\mathcal{P} = 1$ and 2. Note that $b_{\phi S+}$ is given by equation (10) and is independent of $\mathcal{P}$ and b_{rS} is given by equation (13).

4. INTERNAL SOLUTIONS

Here, to simplify the analysis we consider the limit where $\gamma \to \infty$ in equation (11) and $\delta \to 0$ in equation (18). Then, $\zeta_S \to \zeta_m$ and both $\tilde\rho$ and g are unit step functions going to zero at $\zeta_m = \sqrt{2}$. Also, $\overline{u}_r = \langle u_r \rangle$ and $\tilde\Sigma = \sqrt{2}$. Thus the above physical condition $\overline{u}_r \ge 3\varepsilon k_\nu = u_0$ implies that $b_{rS} \ge u_0 \zeta_S \mathcal{P}$ from equation (13). We assume k_p and $k_\nu = 1$.

The solutions to equation (17) are $u_r \propto \exp(ik_j\zeta)$ (with $j = 1,\ 2,\ 3$), where

$$\alpha^4\beta^2(k_j^2)^3 + 2\mathcal{P}\alpha^2\beta(k_j^2)^2 + (\alpha^2\beta^2 + \mathcal{P}^2)k_j^2 - 3\beta\mathcal{P}^2 = 0 , \tag{23}$$

is a cubic in k_j^2. The discriminant of the cubic is negative so that there is one real root, k_1^2, and a complex conjugate pair of roots, $k_{2,3}^2$. Because u_r is an even function of ζ we can write

$$u_r = a_1\cos(k_0\zeta) + a_2\cos(k_r\zeta)\cosh(k_i\zeta) + a_3\sin(k_r\zeta)\sinh(k_i\zeta) , \tag{24}$$

where $k_0 = \sqrt{k_1^2}$, $k_r = \mathrm{Re}(\sqrt{k_2^2})$, and $k_i = \mathrm{Im}(\sqrt{k_2^2})$.

The three unknown constants a_1, a_2, a_3 are reduced to two by imposing equation (20). The two are then reduced to one constant by imposing equation (21). The remaining constant is restricted by the condition $b_{\phi S+} \le 0$.

We consider a thin disk, $\varepsilon = h/r = 0.05$, and a viscosity parameter $\alpha = 0.1$. Figure 1 shows the dependences of the surface field components on the average accretions speed

for $\beta = 100$ and two values of $\mathcal{P}$. The $b_{\phi S}$ dependence is given by equation (9) and is independent of $\mathcal{P}$ while the b_{rS} is given by equation (13).

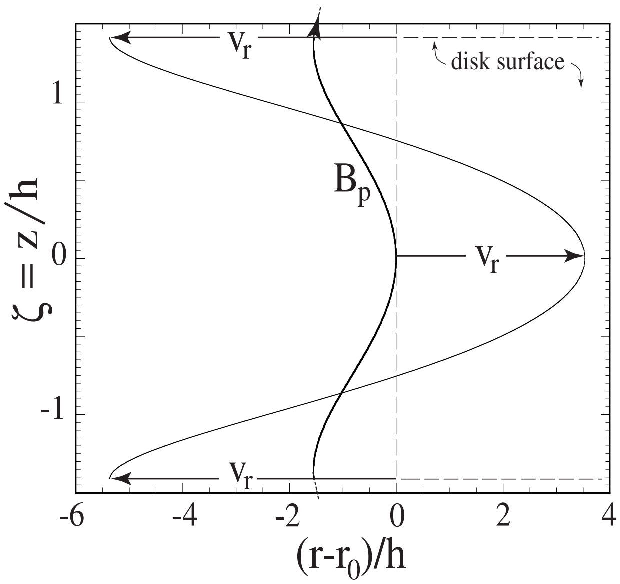

FIG. 2.— Radial flow speed $v_r = -u_r$ (normalized to αc_{s0}) as a function of $\zeta = z/h$ and a sample poloidal (B_r, B_z) magnetic field line for $\beta = 10^2$ and $\mathcal{P} = 1$. Also, r_0 is a reference radius.

Figure 2 shows both the profile of the accretion speed $u_r(\zeta)$ and the shape of the poloidal magnetic field $\mathbf{B}_p$ for $\beta = 100$ and $\mathcal{P} = 1$. Figure 3 shows the profiles of the toroidal magnetic field b_ϕ and the fractional deviation of the toroidal velocity from the Keplerian value, $\delta u_\phi = (v_\phi - v_K)/v_K$. For this case, $\overline{u}_r/u_0 = 1.30$, $b_{\phi S+} = -0.321$, and $b_{rS} = 0.276$. In this case (and a range of others discussed below), we find that there is radial inflow of the top and bottom parts of the disk whereas there is radial *outflow* $u_r < 0$ of the part of the disk around the midplane. Inspection of the flow/field solution shows that the top and bottom parts of the disk lose angular momentum (by the vertical angular momentum flux $rT_{\phi z}$) *both* to (a) magnetic winds or jets from the disk's surfaces *and* to (b) the vertical flow of angular momentum to the midplane part of the disk which flows radially outward.

We find that the flow pattern is the same as in Figure 2 for $10 \leq \beta \leq 200$ and $\mathcal{P} \geq 1$. For larger values of β and $\mathcal{P} = 1$, the flow pattern changes from that in Figure 2 to that in Figure 4 for $\beta = 300$. However, for $\beta = 300$ and $\mathcal{P} = 2$, the flow pattern is again similar to that in Figure 2. For $\beta = 10^3$ and $\mathcal{P} = 3$ the flow pattern is also the same as in Figure 2. For smaller viscosity values, $\alpha \leq 0.03$, and $\beta = 100$ there are multiple channels with for example three layers of radial inflow and two layers of radial outflow in the disk.

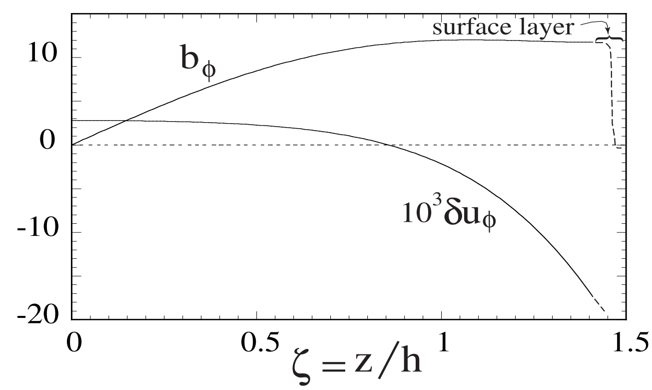

FIG. 3.— Toroidal magnetic field $b_\phi = B_\phi/B_z$ and toroidal velocity $\delta u_\phi = (v_\phi - v_K)/v_K$ (with v_K the Keplerian velocity) for the case where $\beta = 100$ and $\mathcal{P} = 1$. The jump in the toroidal magnetic field at the disk's surface is shown by the dashed line.

Figure 4 shows the profile of the accretion speed $u_r(\zeta)$ and the shape of the poloidal magnetic field $\mathbf{B}_p$ for $\beta = 300$ and $\mathcal{P} = 1$. Figure 5 shows the profiles of the toroidal magnetic field b_ϕ and the fractional deviation of the toroidal velocity from the Keplerian value, $\delta u_\phi = (v_\phi - v_K)/v_K$. For this case, $\overline{u}_r/u_0 = 1.5$, $b_{\phi S+} = -1.59$, and $b_{rS} = 0.318$. Inspection of the flow/field solution shows that the angular momentum flux $rT_{\phi z} > 0$ in the top half of the disk and at the disk's surface this flux goes into an outflow or jet.

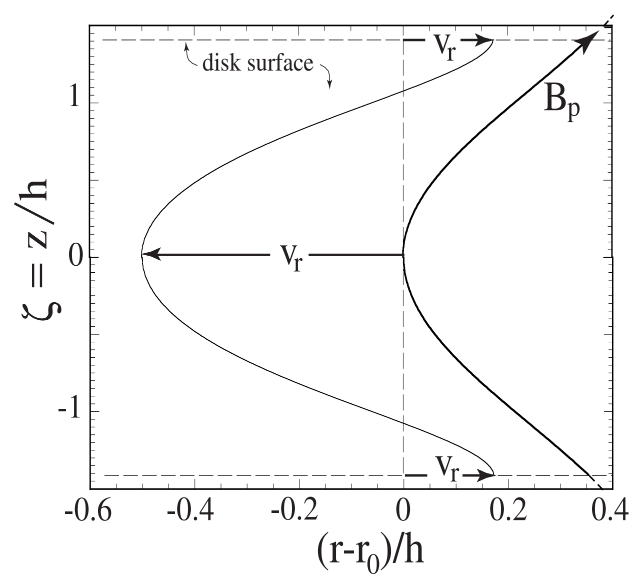

FIG. 4.— Radial flow speed $v_r = -u_r$ (normalized to αc_{s0}) as a function of $\zeta = z/h$ and a sample poloidal (B_r, B_z) magnetic field line for $\beta = 300$ and $\mathcal{P} = 1$. Also, r_0 is a reference radius.

5. EXTERNAL SOLUTIONS

As mentioned in §3, the value of $b_{\phi S+} \leq 0$ is not fixed by the solution for the field and flow inside the disk. Its value can be determined by matching the calculated surface fields b_{rS} and $b_{\phi S+}$ onto an external magnetic wind or jet solution. Stability of the wind or jet solution to current driven kinking is predicted to limit the ratio of the toroidal to axial magnetic field components at the disk's surface

$|b_{\phi S+}|$ to values $\lesssim \mathcal{O}(2\pi r/L_z)$ (Hsu & Bellan 2002; Nakamura, Li, & Li 2007), where L_z is the length-scale of field divergence of the wind or jet at the disk surface. From known wind and jet solutions we estimate $2\pi r/L_z \approx \pi$ (Lovelace, Berk, & Contopoulos 1991; Ustyugova et al. 1999; Ustyugova et al. 2000; Lovelace et al. 2002). Recall that $\overline{u}_r/u_0 - 1 = 2|b_{\phi S+}|/(\alpha\beta\tilde{\Sigma}u_0)$ (from equation 9) is the faction of the accretion power going into the jets or winds. For the mentioned upper limit on $|b_{\phi S+}|$, we find $\overline{u}_r/u_0 - 1 \lesssim \mathcal{O}[2\pi/(\alpha\beta\tilde{\Sigma}u_0)]$. From equation (13) we have $b_{rS} = (\mathcal{P}\zeta_S u_0)(\langle u_r\rangle/u_0)$. Therefore, for $\beta \gg 1$ and $\langle u_r\rangle \approx u_0$, we have $b_{rS} \approx \mathcal{P}\zeta_S u_0$.

The matching of internal and external field/flow solutions has been carried out by Königl (1989) and Li (1995) for the case of self-similar $[B_z(r,0) \sim r^{-5/4}]$ magnetocentrifugally outflows from the disk's surface. These outflows occur under conditions where the poloidal field lines at the disk's surface are tipped relative to the rotation axis by more than 30° which corresponds to $b_{rS} > 3^{-1/2} \approx 0.577$ (Blandford & Payne 1982). The outflows typically carry a significant mass flux. For the internal field/flow solutions discussed in §4 with $\beta \gg 1$, we conclude that b_{rS} is sufficiently large for magnetocentrifugal outflows only for turbulent magnetic Prandtl numbers, $\mathcal{P} \gtrsim 2.7$. Shu and collaborators (e.g., Cai et al. 2008, and references therein) have developed detailed 'X-wind' models which depend on the disk having Prandtl numbers larger than unity. Recent MHD simulations by Romanova et al. (2008) provides evidence of conical or X-wind type outflows for Prandtl numbers ≥ 1.

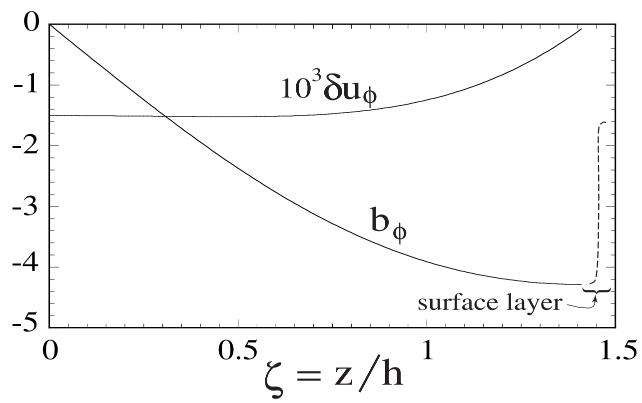

FIG. 5.— Toroidal magnetic field $b_\phi = B_\phi/B_z$ and toroidal velocity $\delta u_\phi = (v_\phi - v_K)/v_K$ (with v_K the Keplerian velocity) for the case where $\beta = 300$ and $\mathcal{P} = 1$. The jump in the toroidal magnetic field at the disk's surface is shown by the dashed line.

For Prandtl numbers say $\mathcal{P} \lesssim 2.7$, the values of b_{rS} are too small for there to be a magnetocentrifugal outflow. In this case there is an outflow of electromagnetic energy and angular momentum from the disk (with little mass outflow) in the form of a magnetically dominated or 'Poynting flux jet' (Lovelace, Wang, & Sulkanen 1987; Lovelace, et al. 2002) also referred to as a 'magnetic tower jet' (Lynden-Bell 1996, 2003). MHD simulations have established the occurrence of Poynting-flux jets under different conditions (Ustyugova et al. 2000, 2006; Kato, Kudoh, & Shibata 2002; Kato 2007). Laboratory experiments have allowed the generation of magnetically dominated jets (Hsu & Bellan 2002; Lebedev et al. 2005).

6. DISCUSSION

A study is made of stationary axisymmetric accretion flows $[v_r(z), v_\phi(z), v_z = 0]$ and the large-scale, weak magnetic field $[B_r(z), B_\phi(z), B_z \approx \text{const}]$ taking into account the turbulent viscosity and diffusivity due to the MRI and the fact that the turbulence vanishes at the surface of the disk as discussed by Bisnovayi-Kogan & Lovelace (2007) and Rothestein & Lovelace (2008). We derive a sixth-order differential equation for the radial flow velocity $v_r(z)$ (equation 17) which depends mainly on the ratio of the midplane thermal to magnetic pressures $\beta > 1$ and the Prandtl number of the turbulence $\mathcal{P}$ = viscosity/diffusivity. Boundary conditions at the disk's surfaces take into account the outflow of angular momentum to magnetic winds or jets and allow for current flow in the highly conducting surface layers. In general we find that there is a radial surface current but no toroidal surface current. The stability of this surface current layer is unknown and remains to be studied. If the layer is unstable to kinking this may cause a thickening of the current layer. We argue that stability of the winds or jets will limit the ratio of the toroidal to axial field at the disk's surface $|b_{\phi S+}|$ to values $\lesssim \pi$. The stationary solutions we find indicate that a weak ($\beta \gg 1$), large-scale field does not diffuse away as suggested by earlier work (e.g., Lubow et al. 1994) which assumed $b_{rS} \geq 3^{-1/2}$.

For a wide range of parameters $\beta > 1$ and $\mathcal{P} \geq 1$, we find stationary *channel* type flows where the flow is radially *outward* near the midplane of the disk and radially *inward* in the top and bottom parts of the disk. Solutions with inward flow near the midplane and outflow in the top and bottom parts of the disk are also found. The solutions with radial outflow near the midplane are of interest for the outward transport of chondrules in protostellar disks from distances close to the star (~ 0.05 AU) (where they are melted and bombarded by high energy particles) to larger distances (> 1 AU) where they are observed in the Solar system. Outward transport of chondrules from distances ~ 0.05 AU to > 1 AU by an X-wind has been discussed by Shu et al. (2001).

The flow/field solutions found here in a viscous/diffusive disk and are different from the exponentially growing channel flow solutions found by Goodman and Xu (1994) for an MRI in an ideal MHD unstable shearing box. Channel solutions in viscous/diffusive disks were found earlier by Ogilvie & Livio (2001) and by Salmeron, Königl, & Wardle (2007) for conditions different from those considered here. In general we find that the magnitude of the toroidal magnetic field component inside the disk is much larger than the other field components. The fact that the viscous accretion speed is very small, $\sim \alpha\varepsilon c_{s0}$, means that even a small large-scale field can significantly influence the accretion flow. We find that Prandtl numbers larger than a critical value estimated to be 2.7 are needed in order for there to be magnetocentrifugal outflows from the disk's surface. For smaller $\mathcal{P}$, electromagnetic outflows are predicted. Owing to the stability condition, $|b_{\phi S+}| \lesssim \pi$, the fraction of the accretion power going into magnetic outflows or jets is $\lesssim \text{const}\beta^{-1} \sim B_z^2$.

Analysis of the time-dependent accretion of the large-scale $\mathbf{B}-$field is clearly needed to study the amplification of the field and build up of magnetic flux in the inner region

of the disk. One method is to use global 3D MHD simulations (Igumenshchev et al. 2003; Igumenshchev 2008), but this has the difficulty of resolving the very thin highly conducting surface layers of the disk. Another method is to generalize the approach of Lovelace et al. (1994) taking into account the results of the present work. This is possible because the radial accretion time $(r/|\overline{u}_r|)$ is typically much longer than the viscous diffusion time across the disk (h^2/ν).

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